

STUDIES ON MIXING. XXX.*

TURBULENT FLOW IN A SYSTEM WITH AXIAL MIXER
AND RADIAL BAFFLES

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This paper presents a study of the turbulent flow of a Newtonian charge in a cylindrical vessel with flat bottom and with radial baffles at the wall. By the use of a photographic method of traces and of the static probe is determined distribution of the time averaged velocity and of the root mean square of the fluctuation velocity in a stream flowing from the blades of a six-bladed paddle mixer with inclined blades. It is determined that there exists similarity between the radial profiles of the mentioned velocities in a liquid stream between the vessel bottom and the rotating mixer if the charge flows from the mixer toward the vessel bottom. The intensity of turbulence in this stream is practically independent of the axial distance from the rotating mixer.

The rotating mixer is causing an intensive convective flow of the charge which may be described by the use of the field with regard to the time averaged velocity. The mixing effect of the mixer cannot be expressed satisfactorily by use of the time averaged streamlines and thus also the fluctuation part of the flow velocity must be considered. Here an attempt is made for a more complete description of the flow leaving the blades of an axial mixer. The time averaged field and the fluctuation velocities are determined and the properties of this field are considered from the mixing point of view.

The velocity field in the stream leaving the blades of an axial mixer (propeller or paddle bladed mixer with inclined blades) was till now measured experimentally with respect to determination of distribution of the time averaged velocity¹⁻⁴. Theoretical studies as well were limited to determination of the profile of this quantity^{3,5,6} and no quantitative or qualitative conclusions were made as far as the relation of pulsation and the in-time averaged velocity in the system concerns. Only experimental studies of the field of time averaged and fluctuation velocities in the system with a radial mixer (turbine mixer with vertical blades) where the method of constant temperature anemometer⁷⁻⁹ and the photographic method of traces were used^{10,11}. From

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results of these studies a conclusion can be made that values of pulsations and of the time averaged velocities are mutually correlated in such places of the system where an intensive flow of the charge takes place, while in regions of relative calmness, the pulsation contribution is the main source of mixing. This fact is obvious also from the spacial heterogeneity of the system with an axial mixer as concerns the rate of energy dissipation which was proved earlier¹². From the published papers may be concluded that despite the turbulence of the mixed charge is their important characteristics which is usually a condition for the operation to take place in the system¹³⁻¹⁵, for axial mixer types is to this fact paid neither the experimental nor theoretical attention.

The flow pattern in the mixed charge at turbulent flow regime consists of several important parts¹⁶. As concerns the contribution to the transfer of properties in the charge itself, the most important are those regions in which the rate of mechanical energy dissipation is the greatest: region in the closest vicinity of the rotating mixer and in the flow leaving its blades¹². Therefore, the experimental study was made in one of those regions, *i.e.* in the region of the liquid stream between the mixer and the vessel bottom.

EXPERIMENTAL

The experiments were made in a cylindrical vessel with flat bottom (made of organic glass "Perspex") of diameter $D = 290$ mm, with four radial baffles at the wall of width $b = 0.10$. For a charge was used distilled water at $20 \pm 1^\circ\text{C}$. Its height H , when at rest, was equal to the vessel diameter D . The used mixers were six-bladed paddle mixers with blades inclined under the angle 45° ^{4,12,17}. Their relative dimensions were consecutively $d/D = 1/5, 1/4$ and $1/3$. The mixer was placed in the vessel axis at a relative distance $1/4$ between the vessel bottom and the liquid surface at rest and always rotated in such direction so as to force the liquid toward the bottom.

The velocity field was measured in the liquid stream flowing from the blades of the mixer. Two measuring methods were used:

1. In vicinity of the rotating mixer was used the photographic method of traces. In this way the flow pattern was obtained in the vertical plane passing through the mixer axis. The measuring procedure as well as accuracy of obtained results was the same as those given in the previous paper³ of this series.

2. In vicinity of the vessel bottom was used the method of three pressure probes and of a static probe¹⁸. By use of this combination was obtained (by procedure described in the mentioned paper¹⁸) the profile of the time averaged velocity and the profile of the root mean square of the fluctuation velocity component. By use of the static probe were measured the fluctuations of static pressure as a time progression of instantaneous values of this quantity in constant time intervals of 4 seconds. In this way a series of sixty instantaneous values of static pressure was always obtained for each position of the static probe and for the given speed of rotation of the mixer. These values were read off on the inclined manometer with an accuracy $\pm 2.5 \text{ N/m}^2$. From the obtained results of pulsations of static pressure, determined by the manometer connected directly without a damping element with the static probe, was calculated the standard deviation of this quantity which is directly proportional to the mean value of second power of fluctuation component of the velocity. The information concerning the pulsation velocity component from results of the photographic method of traces was then obtained by calculation of the standard deviation from several values of instantaneous velocity determined in the considered position at the same geometrical and physical conditions in the mixed system. Due to the used time-consuming experimental technique, as the "point" was considered the line segment in radial direction of 2 mm length by which was determined the time averaged as well as the fluctuation velocity component like the mean value. It was also necessary to consider the average position of the radial velocity profile in this axial direction as the local velocity value was always assigned to the centre of the

trace of the aluminium particle recorded during photographic exposition. In this way, the distance was obtained from the lower edge of the mixer blades equalling to 13 mm to which the determined velocity profile was assigned. This distance corresponds to the relative distance $h_1/H = 0.172$ between the vessel bottom and the liquid surface at rest. Measurements at the vessel bottom were carried out at the relative distance $h_1/H = 0.035$ between the bottom and the liquid surface of the charge at rest¹⁸.

Comparison of the results obtained by the used methods was made on the basis of measured results of velocity fields in the regions above the rotating mixer¹⁹ where both methods were used. By comparing the determined radial profiles of absolute values as well as of vector directions of the time averaged velocities by statistical analysis, it was determined that the difference between the corresponding values along the profile is not significant. From that follows that in a relatively still, steady and oriented stream — such as in the region above the rotating axial mixer where the liquid is sucked into the mixer — the results obtained by both used methods are identical. In the stream where the turbulence intensity is reaching higher values as in the liquid stream flowing from the mixer, the fluctuation component becomes evident with the method of pressure probes, which means that the determined value of the time averaged component differs the more from this quantity, the greater is the intensity of turbulence^{4,12,18}. The effect of turbulence intensity $(\overline{w'^2})^{1/2}/\overline{w}$ on the value of relative error, which is made if we take into consideration the calculated absolute value of the vector of local velocity calculated from pressure of the three-holes Pitot tube, is given in Table I. It is also obvious from this Table that if the value of turbulence intensity is less than 30%, the mentioned relative error is less than the accuracy of the used experimental technique⁴.

TABLE I
Effect of Turbulence Intensity on the Relative Error of Determination of the Time Averaged Velocity from Measurements Made with the Three-Holes Pitot Tube

$(\overline{w'^2})^{1/2}/\overline{w}$ %	$\Delta\overline{w}/\overline{w}$ %
10	0.5
20	1.8
30	4.2
40	7.0
50	10.5

RESULTS AND DISCUSSION

Field of the Time Averaged Velocity

From the experimental study of velocity field in the stream between the rotating mixer and the vessel bottom, profiles were obtained of axial component of the time averaged velocity vector \overline{w}_{ax} and of direction determining its orientation. The mentioned direction was characterized by use of the angle φ which is defined by relations

$$\varphi \equiv \arccos(\overline{w}_{ax}/\overline{w}), \quad (1a)$$

and

$$\varphi \equiv \arctg(\overline{w}_{ax}/\overline{w}_{rad}), \quad (1b)$$

where the quantity \overline{w} is projection of the time averaged velocity vector into the vertical plane passing through the mixer axis. On basis of Eq. (1a) it was possible to calculate from the known experimentally determined angle φ and quantity \overline{w} the component

of the time averaged velocity \bar{w}_{ax}^4 . By use of Eq. (1b) it was possible to calculate from the known, experimentally determined values of axial and radial component of the time averaged velocity vector the value of angle φ . Since the experimental measurements were made at several rotational speed of the mixer, the quantity \bar{w}_{ax} was expressed in a dimensionless form related to the peripheral velocity of the mixer blade according to

$$\bar{W}_{ax} = \bar{w}_{ax}/(\pi dn). \quad (2)$$

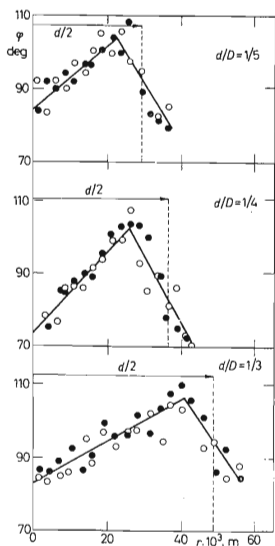


FIG. 1

Radial Profile of Orientation of the Time Averaged Local Velocity Vector — Angle φ ($h_1 = 0.172 H$)

d/D	1/3	1/4	1/5	Point
n, min^{-1}	300	600	900	○
n, min^{-1}	500	900	1 200	●

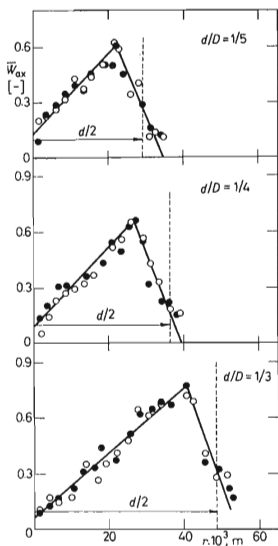


FIG. 2

Radial Profile of Dimensionless Time Averaged Axial Component of Local Velocity Vector ($h_1 = 0.172 H$)

d/D	1/3	1/4	1/5	Point
n, min^{-1}	300	600	900	○
n, min^{-1}	500	900	1 200	●

Radial profiles of angle φ for distance $h_1 = 0.172H$ from the vessel bottom are given in Fig. 1, radial profiles of quantity \bar{W}_{ax} for the same distance from the vessel bottom in Fig. 2. The corresponding radial profiles in the ray located at the vertical distance $h_1 = 0.035H$ are given in Figs 5 and 6 of the cited paper of this series¹⁸.

The profiles of orientation dependence of the time averaged velocity vector in the vertical plane on the radial coordinate in vicinity of the mixer, present a predominant axial component with regard to the radial component. Interesting is the fact that with increasing value of the coordinate r the quantity φ changes continuously — at first it increases and then, from value r_{max} decreases. In vicinity of the point r_{max} , where $\varphi > 90^\circ$, the velocity component \bar{w}_{rad} is oriented toward the system axis, i.e. a certain concentration of the stream takes place which is caused by flow through the rotor region of the mixer¹², where — due to suction effect of the mixer — the stream is drawn into the region above the mixer and where it flows at a considerably greater radial distance from the axis than is the mixer diameter. However, in the descending part of the radial profile of angle φ , the flow of charge in the direction from the system axis takes place due to its successive turn at the vessel bottom. This effect is primarily obvious in vicinity of the bottom ($h_1 = 0.035H$) where the ascending part of the profile $\varphi = \varphi(r)$ practically disappears and where across the considered cross-section the quantity φ successively decreases to zero in the same way as the flow changes its direction because of the bottom. In Table II are given parameters of regression curves calculated by the least square method from the results of experiments together with estimates of their correlation coefficients.* From this Table it

TABLE II

Parameters of Dependence of the Velocity Vector Orientation (Angle φ) on Radial Coordinate r

h_1/H	d/D	$\varphi = \varphi(r)$	Correlation coefficient	r_{max} m
0.035 ^a	1/3	$\varphi = 0.616r^{-2.26}$	—	$3.5 \cdot 10^{-2}$
0.172	1/3	$\varphi = 566r + 83.3$	0.972	$4.05 \cdot 10^{-2}$
0.172	1/3	$\varphi = -1.440r + 164.4$	0.912	—
0.172	1/4	$\varphi = 1.130r + 73.9$	0.984	$2.55 \cdot 10^{-2}$
0.172	1/4	$\varphi = -1.840r + 149.6$	0.918	—
0.172	1/5	$\varphi = 862r + 84.4$	0.981	$2.25 \cdot 10^{-2}$
0.172	1/5	$\varphi = -1.700r + 142.7$	0.914	—

^a $r < 3.5 \cdot 10^{-2}; 11 \cdot 10^{-2} \text{ m}$, $\varphi_{max} 87^\circ$

* For flow in vicinity of the bottom ($h_1 = 0.035H$) the dependence $\varphi = \varphi(r)$ was approximated by a power function.

is obvious that position of the maximum on the profile $\varphi = \varphi(r)$, characterized by the coordinate r_{\max} , depends on the relative size d/D of the mixer and vessel as well as on the relative distance h_1/H from the vessel bottom.

The radial profiles of the quantity \overline{W}_{ax} have a similar course as the function $\varphi = \varphi(r)$. With the increasing value of coordinate r , the axial velocity component firstly increases and after the ascending part follows the descending part of the profile where – with the increasing radial coordinate – the quantity \overline{W}_{ax} decreases. From

Fig. 2 as well as from the cited paper¹⁸ (where are given profiles $\overline{W}_{ax} = \overline{W}_{ax}(r)$ at the vertical distance from the bottom $h_1 = 0.035H$ for the mixer sizes $d/D = 1/3$ and $1/4$) follows, that the flow velocity of charge is directly proportional to the

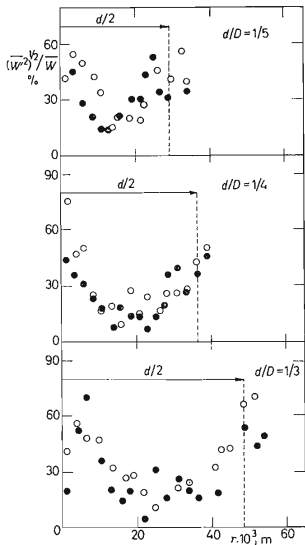


FIG. 3
Radial Profile of Turbulence Intensity ($h_1 = 0.172H$)

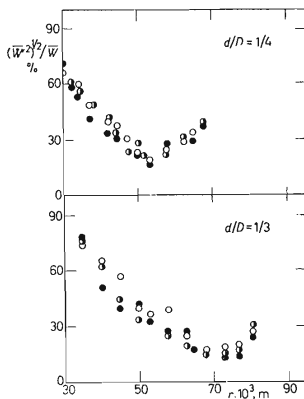


FIG. 4
Radial Profile of Turbulence Intensity ($h_1 = 0.035H$)

d/D	$1/3$	$1/4$	$1/5$	Point	$d/D = 1/3$ n, min^{-1}	$d/D = 1/4$ n, min^{-1}	Point
n, min^{-1}	300	600	900	○	300	600	○
n, min^{-1}	500	900	1 200	◐	500	900	◐
				●	575	1 100	●

rotational speed of the mixer. In Table III are given parameters of regression straight lines calculated by the least square method from the results of experiments together with estimates of their correlation coefficients. From this Table as well as from the given corresponding figures it is obvious that both branches of the profile may be expressed by an equation of straight line. This is in agreement with theoretical considerations about the shape of profile $\bar{W}_{ax} = \bar{W}_{ax}(r)$ in the stream at the exit from the blades of the rotating axial mixer^{3,6}. But with the increasing distance from the mixer, the velocity profile changes; by transverse pulsations in this stream the velocities in the perpendicular direction start to equalize which causes smoothing of the descending part of the profile from the hyperbolic to the straight line profile and also the slope of the ascending part of the profile is less. This quantity, as is obvious from Table III, is smaller than would be the theoretical value in the stream at the exit from the mixer blades (equal $2/d$)³ and its value decreases in direct proportion to the increasing vertical distance from the lower edge of the blade tips. Besides, although the course of dependence $\bar{W}_{ax} = \bar{W}_{ax}(r)$ is being kept over the whole studied stream *i.e.* also in vicinity of the bottom, a shift takes place of the mentioned profile in the radial distance toward the wall because due to the bottom a turn of flow in this direction takes place. In vicinity of the bottom along the system axis appears the so called "dead region" where even the reversed flow⁴ toward the mixer takes place.

Field of Fluctuation Velocity

From the results of measured velocity fields by both used methods the estimate was calculated of root mean square of the fluctuation velocity $(\overline{w'^2})^{1/2}$. From the results of measurements of instantaneous local velocities by the photographic method of traces was for the given place calculated beside its arithmetic mean also its standard deviation which was considered to be the estimate of quantity $(\overline{w'^2})^{1/2}$. This quantity was calculated for the given place from ten values of instantaneous velocities. The mean value of the second power of fluctuation velocity w'^2 may be estimated from fluctuations of static pressure registered by the static probe. For the case of isotropy or at least of the local isotropy Batchelor²⁰ derived equation relating the root mean square of fluctuation component of static pressure and the mean value of second power of fluctuation component of velocity, which is

$$(\overline{p'_{st}})^{1/2} \approx 0.70 \overline{w'^2} \quad (3)$$

Since the quantity $(\overline{p'_{st}})^{1/2}$ equals to the standard deviation of static pressure it is possible from the determined pulsations to estimate quantitatively pulsations of local velocity expressed by quantity $(\overline{w'^2})^{1/2}$. The estimate of the second power of fluctuation velocity component from fluctuations of the static pressure depends on the inertia of the probe by which the fluctuations of static pressure are indicated. By decreasing

the inertia *i.e.* by shortening of the connecting tubes and by increasing their diameter very fast pulsations with higher frequency may be also recorded while greater pulsations of smaller frequency are recorded even at greater inertia of the measuring instrument. By the use of the static probe was thus determined a certain estimate of second power of fluctuation velocity which was the greater than the real value, the greater was inertia of the used instrument.

Beside this quantity it was possible from results of measurements obtained by the photographic method of traces to calculate the quantity $(\overline{\varphi'^2})^{1/2}$ characterizing pulsations of the component of orientation of local velocity vector in the

TABLE III

Parameters of Dependence of the Dimensionless Axial Component of the Time Averaged Local Velocity \overline{W}_{ax} on Radial Coordinate r

h_1/H	d/D	$\overline{W}_{ax} = \overline{W}_{ax}(r)$	Correlation coefficient	Range of r m	r_{max} m
0.035	1/3	$\overline{W}_{ax} = 5.52r - 0.1$	0.972	$\langle 1.9 \cdot 10^{-2}; 5.9 \cdot 10^{-2} \rangle$	—
0.035	1/3	$\overline{W}_{ax} = -4.29r + 0.474$	0.926	$\langle 5.9 \cdot 10^{-2}; 11.1 \cdot 10^{-2} \rangle$	—
0.035	1/4	$\overline{W}_{ax} = 5.5r - 0.095$	0.981	$\langle 1.8 \cdot 10^{-2}; 4.9 \cdot 10^{-2} \rangle$	—
0.035	1/4	$\overline{W}_{ax} = -4.43r + 0.42$	0.941	$\langle 4.9 \cdot 10^{-2}; 10.2 \cdot 10^{-2} \rangle$	—
0.172	1/3	$\overline{W}_{ax} = 17.4r + 0.062$	0.988	—	$4.05 \cdot 10^{-2}$
0.172	1/3	$\overline{W}_{ax} = -78.8r + 4.153$	0.931	—	—
0.172	1/4	$\overline{W}_{ax} = 21.4r + 0.089$	0.976	—	$2.68 \cdot 10^{-2}$
0.172	1/4	$\overline{W}_{ax} = -55.2r + 2.172$	0.922	—	—
0.172	1/5	$\overline{W}_{ax} = 21.7r + 0.133$	0.972	—	$2.22 \cdot 10^{-2}$
0.172	1/5	$\overline{W}_{ax} = -50.0r + 1.773$	0.912	—	—

TABLE IV

Average Values of Turbulent Characteristics in the Stream between the Axial Mixer and the Vessel Bottom

h_1/H	d/D	$[(\overline{w'^2})^{1/2}/\overline{w}]_{av}$ %	$(\overline{\varphi'^2})_{av}^{1/2}$ deg
0.035	1/3	29.0	—
0.035	1/4	36.2	—
0.172	1/3	36.6	16.0
0.172	1/4	30.0	13.6
0.172	1/5	33.6	14.4

vertical plane passing through the axis of the mixed system. In Fig. 3 and 4 are given radial profiles of the turbulence intensity $(\overline{w'^2})^{1/2}/\overline{w}$ in both measured distances above the bottom at the used relative mixer size d/D . In Table IV are given mean values of turbulent characteristics calculated as their arithmetic means from results of all experiments in the given flow cross-sectional area.

It follows from calculations of radial profiles of quantity $(\overline{w'^2})^{1/2}/\overline{w}$, that their shape depends on relative size d/D of the mixer and vessel and further that their variance is greater in vicinity of the mixer than in vicinity of the bottom because in vicinity of the bottom the effect of the mixer as a source of pulsations is decreased. But in both considered regions the effect of rotational speed of the mixer on quantity $(\overline{w'^2})^{1/2}/\overline{w}$ may be considered as insignificant which may lead to the conclusion that not only the time averaged velocity but also the pulsation velocity component of flow of the mixed charge is proportional to the first power of rotational speed of the mixer. The radial dependences $(\overline{w'^2})^{1/2} = (\overline{w'^2})^{1/2}(r)$ and $(\overline{\varphi'^2})^{1/2} = (\overline{\varphi'^2})^{1/2}(r)$ were found inexpressive with respect of the variance. These quantities may be thus considered constant in the given cross-sectional area of flow between the mixer and bottom. From Table III follows that the quantity $(\overline{\varphi'^2})_{av}^{1/2}$ is practically independent on the relative mixer size and vessel but its value indicates significant fluctuations in radial direction with respect of the magnitude of the radial component of the time averaged velocity vector in vicinity of the maximum of the profile $\varphi = \varphi(r)$. The average turbulence intensity $[(\overline{w'^2})^{1/2}/\overline{w}]_{av}$ across the whole flow ray also does not depend significantly on the relative size of the mixer but it also does not depend either on the situation of the cross-section in this stream with respect of the mixer: in vicinity of the mixer as well as in vicinity of the bottom are values of this quantity close one to the other, from which follows that the decay of turbulence in the considered stream is directly proportional to the decrease of velocity in this stream. From this fact also follows that the mixing length in the region between the mixer and bottom is practically constant *i.e.* that the mixing intensity in micro-scale may be in this part of the mixed system considered constant quantity. This fact was earlier proved indirectly by experiments for geometrically similar systems in a study of processes in a mixed charge such as for inst. homogenisation of miscible liquids or solution of a solid phase when the rate of these processes in the considered region across the whole volume was found constant^{15,21}. Thus a jet streaking from the blades of a mixer toward the vessel bottom can be considered an isotropic region from the point of view of turbulent characteristics here considered *i.e.* from the point of view of the turbulence intensity and pulsations of orientations of local velocity vector.

LIST OF SYMBOLS

- b width of radial baffle [m]
 D vessel diameter [m]

d	mixer diameter [m]
H	liquid height from the vessel bottom at rest [m]
h_1	vertical distance between the vessel bottom and the given cross-sectional area in the stream between the rotating mixer and the vessel bottom [m]
n	rotational speed of the mixer [s^{-1}]
p'_{st}	fluctuation component of static pressure [$N m^{-2}$]
r	radial distance from the axis of symmetry of the mixed system [m]
r_{max}	position on the radial profile where the dependent quantity has its maximum value [m]
\bar{w}	projection of the time averaged local velocity vector into the vertical plane passing through the axis of the mixed system [$m s^{-1}$]
\bar{w}_{ax}	axial component of the time averaged component of local velocity vector [$m s^{-1}$]
\bar{w}_{rad}	radial component of the time averaged local velocity vector [$m s^{-1}$]
w'	fluctuation component of local velocity vector [$m s^{-1}$]
\bar{w}'_{ax}	dimensionless value of the time averaged axial component of local velocity vector
ρ	density of the mixed charge [$kg m^{-3}$]
φ	angle between the time averaged local velocity vector and the horizontal direction [deg]
φ'	fluctuation component of quantity φ [deg]
av	index denoting mean value over the given cross-section in the stream between the mixer and the vessel bottom
$\Delta\bar{w}$	error in determination of the time averaged local velocity vector caused by neglecting the fluctuation velocity component [$m s^{-1}$]

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